AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Unit Further Pure 4

Wednesday 23 May 2018   Morning   Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 The matrix $\mathbf{M} = \begin{bmatrix} -1 & 0 & 3 \\ 1 & 4 & 2 \\ 5 & 1 & -1 \end{bmatrix}$ and the matrix $\mathbf{N}$ is a 3 by 3 square matrix such that $\det \mathbf{N}^{-1} = 17$.

A three dimensional shape, S, with volume $8.5\,\text{cm}^3$ is mapped onto S' by the transformation with matrix $\mathbf{MN}^2$. Find the volume of S'.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 1

2 Three non-singular square matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are such that

$$\mathbf{BA} = \mathbf{C}$$

The matrix $\mathbf{B}$ represents a reflection in the $x = y$ plane and $\mathbf{C} = \begin{bmatrix} 0.28 & -0.96 & 0 \\ -0.96 & -0.28 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Give a full geometric description of the single transformation represented by the matrix $\mathbf{A}$. Give the value of any angle involved to the nearest degree.

[5 marks]

QUESTION PART REFERENCE

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3 The matrix $\mathbf{P} = \begin{bmatrix} -3 & -k & 2 \\ -4 & k & -2 \\ 1 & -1 & -1 \end{bmatrix}$, where k is a constant.

(a) Find the determinant of $\mathbf{P}$, in terms of k.

[1 mark]

(b) Given that $\mathbf{P}$ is a non-singular matrix, find $\mathbf{P}^{-1}$ in terms of k.

[5 marks]

QUESTION PART REFERENCE

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4 The line l has equation $x - 2 = 1 - y = \frac{2z - 3}{4}$ and the plane Π has equation $2x + py + z = 10$, where p is a constant. Given that the **acute** angle between the line and the plane is $30°$, find the possible values of p.

[6 marks]

QUESTION PART REFERENCE

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5 Three planes have equations

$$2x + 3y - z = 8$$

$$3x + ky + z = -6$$

$$4x + 6y + (k+1)z = -4k + 4$$

(a) In the case where the three planes intersect in a single point, find the coordinates of the point in terms of k.

[5 marks]

(b) In the case where the three planes do not meet in a single point, find the two possible values of k.

[3 marks]

(c) For each value of k found in part **(b)**, find the number of solutions of the given system of equations, fully justifying your answers, **and** give a geometrical interpretation in each case in relation to the planes.

[4 marks]

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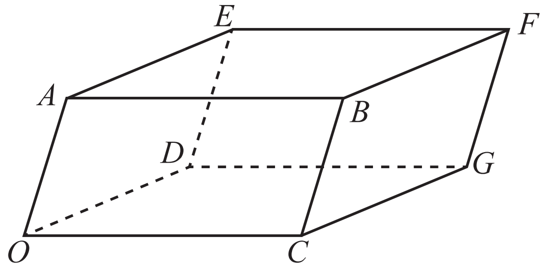
6 The diagram shows the parallelepiped with vertices O, A, B, C, D, E, F and G.



Points A, B, C and D have position vectors

$$\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \mathbf{d} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

respectively, relative to the origin O.

(a) Find the volume of the parallelepiped.

[2 marks]

(b) (i) Determine a vector equation for the plane $BCDE$ in the form $\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$.

[3 marks]

(ii) Determine a vector equation for the plane $ABGD$ in the form $\mathbf{r}.\mathbf{n} = k$.

[3 marks]

(iii) Find an equation for the line of intersection of the planes $BCDE$ and $ABGD$, giving your answer in the form $(\mathbf{r} - \mathbf{p}) \times \mathbf{q} = \mathbf{0}$.

[3 marks]

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7 The matrix $\mathbf{M} = \begin{bmatrix} x & y^2 & y+z \\ y & x^2 & x+z \\ x+y & 2y^2 & z \end{bmatrix}$.

(a) Express $\det \mathbf{M}$ as the product of linear factors.

[6 marks]

(b) In the case where $x = 2$, $y = 3$ and $\mathbf{M}$ is a singular matrix, find the value of z.

[2 marks]

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8 The transformation S is a shear and maps the point (x, y) to (x', y') such that $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 3 \\ b & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ where a, b and c are constants.

(a) Write down the value of $ac - 3b$.

[1 mark]

(b) The image of $(-2, 1)$ under S is $(-5, 4)$.

(i) Find the values of a, b and c.

[4 marks]

(ii) Find an equation of the invariant line of S which passes through the point $(0, 2)$.

[3 marks]

QUESTION PART REFERENCE

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9 For real numbers p and q with $q \neq 0$ and $q \neq \pm p$ the matrix $\mathbf{M} = \begin{bmatrix} -p & q-p \\ p+q & p \end{bmatrix}$.

(a) Find the eigenvalues of $\mathbf{M}$.

[3 marks]

(b) (i) Show that $\begin{bmatrix} q-p \\ p+q \end{bmatrix}$ is an eigenvector of $\mathbf{M}$, and state the associated eigenvalue.

[2 marks]

(ii) Show that a second eigenvector of $\mathbf{M}$ is independent of p and q.

[2 marks]

(c) (i) Find matrices $\mathbf{U}$ and $\mathbf{D}$ such that $\mathbf{M} = \mathbf{UDU}^{-1}$.

[2 marks]

(ii) Hence, in the case when n is an odd number, show that $\mathbf{M}^n = q^{n-1}\mathbf{M}$.

[6 marks]

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END OF QUESTIONS